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Two variations of certainty control are presented. The original certainty control formulation produced a shrinking sphere about the predicted impact point with the surface being a function of estimated error. The first variation of this formulation introduces a control effectiveness ratio to regulate thrusting time. The second formulation replaces the shrinking sphere with a shrinking ellipsoid; if the predicted miss is inside or touching the ellipsoid, thrusting is not necessary. Variational performances in lateral thrusting are examined for a hypervelocity, exoatmospheric, orbital vehicle in the final 30 seconds of flight while it is attempting to intercept a boosting missile. Neither variation simultaneously reduces maneuvering cost and miss distance, indicating that the original formulation effectively uses the state error estimates.

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S. Alfano

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Two Variations of Certainty Control

Salvatore Alfano*
U.S. Air Force Academy,
Colorado Springs, Colorado 80840

Introduction

CERTAINTY control[1] enhances interceptor performance by using a terminal guidance law that incorporates the dynamics of the interceptor *and target* plus the error knowledge of their estimates. This is done by constraining the final estimated state to a spherical inequality based on the projected estimate error. The control law reduces intercept maneuvering when the controls associated with cost do not affect state estimate certainty.[2]

Two variations of certainty control are presented in an attempt to further improve interceptor performance: the first uses a control effectiveness ratio to regulate thrusting times; the second changes the certainty control constraint to an ellipsoidal function based on projected estimate error. Conceptually, the second variation produces a shrinking ellipsoid about the predicted impact point with the surface being a function of estimated error; if the predicted miss is inside or touching the ellipsoid, thrusting is not necessary.

Variational performances in lateral thrusting are examined for a hypervelocity, exoatmospheric, orbital vehicle in the final 30 s of flight while it is attempting to intercept a boosting missile. System modeling and measurement processing are identical to those used in Ref. 1. Target tracking is accomplished with a ranging device and line-of-sight sensors for in-plane and out-of-plane measurements. Noise-corrupted data are processed through an eight-state extended Kalman filter (EKF) with serial updates occurring every 0.1 s. Velocity changes are determined by varying impact conditions using splines to reduce computational burdens and allow a solution that lends itself to deterministic techniques.

Optimum Spacing of Corrective Thrusts

Corrective thrusting in the presence of state estimate errors can be optimally spaced to reduce fuel. A control effectiveness

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*Associate Professor, Deputy for Labs and Research, Department of Astronautics. Senior Member AIAA.

ratio,[3] ρ, is established to determine the spacing between thrusts; the ratio directly yields thrust times when control effectiveness is a linear function of time. Control effectiveness is measured by the amount the end conditions vary for a specified control; for this problem, as intercept time decreases, so does the ability of lateral thrusting to vary miss distance. With a ratio of two ($\rho = 2.0$), the corrective thrusting should only occur when the control has half the effect ($1/\rho$) of the previous corrective thrust. If control effectiveness is a near-linear function of time, as is the case for a hypervelocity orbital vehicle, then it will be halved at about half the time to impact since the last thrust. Thrusting will occur at the start of the intercept, at one-half time-to-go, one-fourth time-to-go, one-eighth time-to-go, etc. When spacing is less than the estimator's cycle time (0.1 s for this study), impact is imminent and thrust is terminated.

Certainty Control Formulation with Ellipsoidal Constraint

As stated earlier, if the controls associated with cost do not affect state estimate certainty, then fuel may be conserved by using that certainty to reduce control efforts. When the controls are linked to the estimate certainty, a near-perfect estimate yields the optimal control (certainty equivalence solution[4]), and a poor estimate causes a reduction in control. Certainty control does this by constraining the final estimated states to a spherical inequality based on the projected estimate error. An ellipsoidal inequality is introduced by establishing the cost function,

$$L = \frac{\Delta V_y^2 + \Delta V_z^2}{2} \tag{1}$$

subject to the following constraint:

$$f = \frac{\hat{x}_f^2}{2\sigma_{xf}^2} + \frac{\hat{y}_f^2}{2\sigma_{yf}^2} + \frac{\hat{z}_f^2}{2\sigma_{zf}^2} - \frac{K}{2} \le 0 \tag{2}$$

where ΔV_y and ΔV_z are the interceptor's velocity changes, and K is the constraint weighting factor. Time-to-go t_{go} is used as a third control parameter to minimize miss distance but does not explicitly appear in the cost function. The final state estimates $(\hat{x}_f, \hat{y}_f, \hat{z}_f)$ and their deviations $(\sigma_{xf}, \sigma_{yf}, \sigma_{zf})$ are determined by running the filter forward to predicted impact time without measurement or control updates and then representing their time history with splines:

$$x_s = A_x t_{go}^3 + B_x t_{go}^2 + C_x t_{go} + D_x \tag{3}$$

$$y_s = A_y t_{go}^3 + B_y t_{go}^2 + C_y t_{go} + D_y \tag{4}$$

$$z_s = A_z t_{go}^3 + B_z t_{go}^2 + C_z t_{go} + D_z \tag{5}$$

$$\hat{x}_f = x_s \tag{6}$$

$$\hat{y}_f = y_s - \Delta V_y t_{go} \tag{7}$$

$$\hat{z}_f = z_s - \Delta V_z t_{go} \tag{8}$$

$$\sigma_{xf} = A_{\sigma x} t_{go}^3 + B_{\sigma x} t_{go}^2 + C_{\sigma x} t_{go} + D_{\sigma x} \tag{9}$$

$$\sigma_{yf} = A_{\sigma y} t_{go}^3 + B_{\sigma y} t_{go}^2 + C_{\sigma y} t_{go} + D_{\sigma y} \tag{10}$$

$$\sigma_{zf} = A_{\sigma z} t_{go}^3 + B_{\sigma z} t_{go}^2 + C_{\sigma z} t_{go} + D_{\sigma z} \tag{11}$$

Conceptually, the constraint produces a deviation ellipsoid about the predicted impact point. If the predicted miss is inside or touching the ellipsoid, then thrusting is not necessary. If the predicted miss is outside the ellipsoid, then minimum thrusting is determined to bring the miss to the ellipsoid's surface. Sensor inaccuracies will cause the predicted impact point (ellipsoid center) to jitter with each measurement; chasing this point (i.e., attempting a zero-miss solution) results in counterproductive maneuvering. As the estimates improve, the constraint tightens and the ellipsoid shrinks, along with predicted miss distance. The spline representations allow this stochastic problem to be solved in a deterministic fashion[7] by adjoining the constraint to the cost function to form the Hamiltonian

$$H = L + \lambda f \tag{12}$$

where λ is the Lagrangian multiplier. The partials of H with respect to the controls must equal zero:

$$\frac{\partial H}{\partial \Delta V_y} = \Delta V_y - \frac{\lambda \hat{y}_f t_{go}}{\sigma_{yf}^2} = 0 \tag{13}$$

$$\frac{\partial H}{\partial \Delta V_z} = \Delta V_z - \frac{\lambda \hat{z}_f t_{go}}{\sigma_{zf}^2} = 0 \tag{14}$$

$$\frac{\partial H}{\partial t_{go}} = \lambda \left(\frac{\hat{x}_f \dot{\hat{x}}_f}{\sigma_{xf}^2} + \frac{\hat{y}_f \dot{\hat{y}}_f}{\sigma_{yf}^2} + \frac{\hat{z}_f \dot{\hat{z}}_f}{\sigma_{zf}^2} - \frac{\hat{x}_f^2 \dot{\sigma}_{xf}}{\sigma_{xf}^3} - \frac{\hat{y}_f^2 \dot{\sigma}_{yf}}{\sigma_{yf}^3} - \frac{\hat{z}_f^2 \dot{\sigma}_{zf}}{\sigma_{zf}^3} \right) = 0 \tag{15}$$

with

$$\dot{\hat{x}}_f = 3A_x t_{go}^2 + 2B_x t_{go} + C_x \tag{16}$$

$$\dot{\hat{y}}_f = 3A_y t_{go}^2 + 2B_y t_{go} + C_y - \Delta V_y \tag{17}$$

$$\dot{\hat{z}}_f = 3A_z t_{go}^2 + 2B_z t_{go} + C_z - \Delta V_z \tag{18}$$

$$\dot{\sigma}_{xf} = 3A_{\sigma x} t_{go}^2 + 2B_{\sigma x} t_{go} + C_{\sigma x} \tag{19}$$

$$\dot{\sigma}_{yf} = 3A_{\sigma y} t_{go}^2 + 2B_{\sigma y} t_{go} + C_{\sigma y} \tag{20}$$

$$\dot{\sigma}_{zf} = 3A_{\sigma z} t_{go}^2 + 2B_{\sigma z} t_{go} + C_{\sigma z} \tag{21}$$

Equations (2), (13), (14), and (15) constitute four equations with four unknowns, which can be reduced to two equations and two unknowns using Eqs. (7) and (8). Substituting Eq. (7) into Eq. (13) yields

$$\Delta V_y = \frac{\lambda y_s t_{go}}{\sigma_{yf}^2 + \lambda t_{go}^2} \tag{22}$$

$$\hat{y}_f = \frac{y_s \sigma_{yf}^2}{\sigma_{yf}^2 + \lambda t_{go}^2} \tag{23}$$

In a similar manner, substituting Eq. (8) into Eq. (14) yields

$$\Delta V_z = \frac{\lambda z_s t_{go}}{\sigma_{zf}^2 + \lambda t_{go}^2} \tag{24}$$

$$\hat{z}_f = \frac{z_s \sigma_{zf}^2}{\sigma_{zf}^2 + \lambda t_{go}^2} \tag{25}$$

Equations (2) and (15) can now be solved in terms of λ and t_{go}, with ΔV_y and ΔV_z determined afterward from Eqs. (22) and (24). The parameters λ and t_{go} can be found by numerical techniques using the Jacobian:

$$[J] \begin{bmatrix} \mathrm{d}t_{go} \\ \mathrm{d}\lambda \end{bmatrix} = \begin{bmatrix} -f_1 \\ -f_2 \end{bmatrix} \tag{26}$$

$$f_1 = \frac{\hat{x}_f^2}{2\sigma_{xf}^2} + \frac{\hat{y}_f^2}{2\sigma_{yf}^2} + \frac{\hat{z}_f^2}{2\sigma_{zf}^2} - \frac{K}{2} \tag{27}$$

$$f_2 = \lambda \left(\frac{\hat{x}_f \dot{\hat{x}}_f}{\sigma_{xf}^2} + \frac{\hat{y}_f \dot{\hat{y}}_f}{\sigma_{yf}^2} + \frac{\hat{z}_f \dot{\hat{z}}_f}{\sigma_{zf}^2} - \frac{\hat{x}_f^2 \dot{\sigma}_{xf}}{\sigma_{xf}^3} - \frac{\hat{y}_f^2 \dot{\sigma}_{yf}}{\sigma_{yf}^3} - \frac{\hat{z}_f^2 \dot{\sigma}_{zf}}{\sigma_{zf}^3} \right) \tag{28}$$

$$J = \begin{vmatrix} \dfrac{\partial f_1}{\partial t_{go}} & \dfrac{\partial f_1}{\partial \lambda} \\ \dfrac{\partial f_2}{\partial t_{go}} & \dfrac{\partial f_2}{\partial \lambda} \end{vmatrix} \tag{29}$$

If the states are perfectly known, the $\dot{\sigma}$ terms will be zero and the constraint of Eq. (2) will only be satisfied with a predicted miss of zero; the ellipsoidal certainty control equations reduce to the certainty equivalence optimal-control formulation.[1] If the estimate is poor, the σ terms will be large and the inequality constraint of Eq. (2) will result in very little, if any, change in velocity.

Computer Simulation

A head-on, 10-deg out-of-plane intercept is examined with a time-to-go of 30 s. The interceptor is initially traveling at 12 km/s at an altitude of 750 km with a lateral acceleration range of 3-60 m/s^2 in each axis. The booster's initial acceleration is 3.15788 m/s^2, with a unitized mass flow rate of 0.01579 s^{-1}.

A time lag of 0.1 s is used when computing velocity changes to account for measurement processing, controller processing, and thruster response. Target acquisition is assumed to take 3 s; thrusting is not permitted during this time. This simulation, written in Fortran 77 to run on a VAX 3600, generates 200 Monte Carlo runs per case.

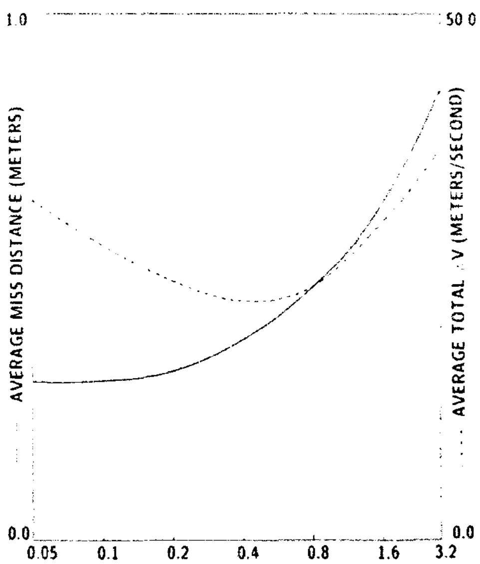

Fig. 1 Performance of certainty control for head-on, 10-deg out-of-plane intercept.

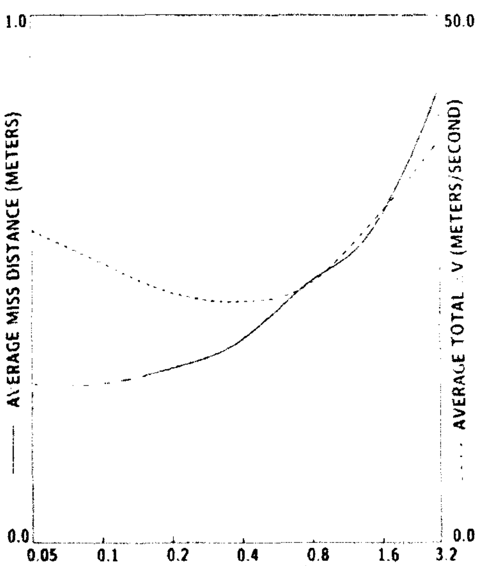

Fig. 2 Performance of certainty control for head-on, 10-deg out-of-plane intercept with $\rho = 1.1$.

Table 1 Minimum ΔV performance for 10-deg out-of-plane intercept

Control strategy	Control multiplier, K	ΔV, m/s	Miss distance, m
Certainty control (original formulation)	0.472	23.13	0.404
Certainty control ($\rho = 1.1$)	0.383	23.26	0.383
Certainty control (ellipsoidal constraint)	1.457	21.88	0.476

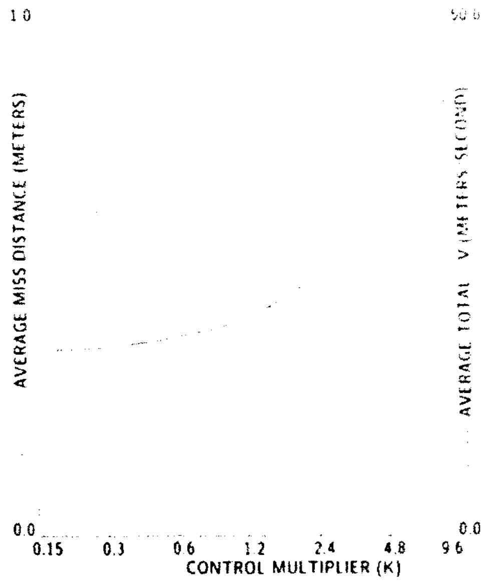

Fig. 3 Performance of ellipsoidal constraint control for head-on, 10-deg out-of-plane intercept.

Results

The original certainty control performance serves as a basis of comparison for the variations, with Fig. 1 showing the relationships of miss distance and total ΔV to the constraint parameter K. Figure 2 illustrates the relationships when the thrusting times are limited by a control effectiveness ratio of 1.1, and Fig. 3 shows the effect of altering the control constraint from spherical to ellipsoidal.

The minimum ΔV performances of all three control strategies are found in Table 1. A control effectiveness ratio of 1.1 slightly reduces the miss distance and increases the total ΔV, whereas larger ratios degrade performance for this particular intercept. Reformulating the control law using an ellipsoidal constraint reduces the total ΔV, but sacrifices some accuracy. Although they are inconclusive, these results indicate that the original formulation of certainty control effectively uses the state deviations to minimize maneuvering costs while maintaining a high level of accuracy.

Conclusions

In this Note, two variations of certainty control were examined to determine their capability to minimize lateral velocity changes of a hypervelocity orbital vehicle in a head-on, 10-deg out-of-plane intercept. The first variation used a control effectiveness ratio to regulate thrusting times; the second changed the spherical constraint function to an ellipsoidal one. Neither variation simultaneously reduced maneuvering cost and miss distance when compared to the original formulation of certainty control, indicating that the original formulation of certainty control effectively uses the state deviations to minimize maneuvering costs while maintaining a high level of accuracy. The ellipsoidal variation best demonstrated a tradeoff in accuracy to reduce maneuvering costs, a choice to be made based on mission constraints. An area of further research is to find a way to make better use of the state deviations in formulating the control law, i.e., reducing cost while further improving accuracy.

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